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Numerical Solutions to Hall Magnetohydrodynamic Equations near an X-type Magnetic Neutral Line

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ABSTRACT: The Hall Magnetohydrodynamic (MHD) model is a new paradigm for describing fast magnetic reconnection processes in space and laboratory plasmas. Current sheets form and store enormous amounts of magnetic energy at X-type magnetic neutral points, which is released as magnetic storms when the sheets break up. The fast magnetic reconnection process impacts solar flares and Earth's geomagnetic sub-storms, which affect global weather. The fast magnetic reconnection process also influences fusion reactors, which may be used as a future energy source. Numerical analysis of approximate solutions to the Hall MHD equations at X-type magnetic neutral points offer these solutions further credence and enhance our understanding of the aforementioned physical phenomena. The solutions to the Hall MHD equations must obey compatibility conditions like incompressibility and an exact invariant. The result, achieved through asymptotic analysis, correctly fits physical laws up to $O(t)$. On the other hand, the exact invariant can be used to refine the asymptotics to any order of accuracy. The asymptotic solutions fully satisfy the plasma incompressibility condition. The most notable of changes between the MHD solutions and the Hall MHD solutions is the ability of the Hall term to prevent the finite-time singularity that appears in MHD. This leads to the prevention of the current density blow-up at long times in Hall MHD.

KEYWORDS: plasma physics, magnetic reconnection, nonlinear differential equations, Taylor's theorem

INTRODUCTION

Magnetic reconnection occurs as a result of non-ideal effects in Ohm's law. Physically, the close encounter of magnetic field lines causes the magnetic field gradients to become locally strong, thus enhancing the formally weak non-ideal process in Ohm's law. Hence, reconnection is a localized process. Reconnection allows the rapid conversion of magnetic energy into kinetic energy [1]. One case of particular interest is magnetic reconnection at X-type neutral points. These points are essentially where hyperbolic magnetic fields meet and create magnetic neutral lines within the plasma flow in the form of an X-point [13].

A thin neutral current sheet is then formed when plasma collapses near the neutral line of the applied magnetic field. In resistive magnetohydrodynamics (MHD) the ion inflow is the only means to transport magnetic flux into the reconnection layer. As the magnetic flux continually accumulates in the region of the neutral sheet, the total current and the sheet width increase until large magnetic pressure gradients develop, which inhibit the ion inflow [15]. The Hall effect [16] can overcome this [4,7], thanks to the decoupling of electrons from ions on length scales below the ion skin depth d_i. If the reconnection layer width is less than d_i, the electron inflow can keep transporting the magnetic flux into the reconnection layer and hence reduce the flux pile-up. Previous numerical work [12, 10, 5, 17] indicated that the dissipation in Hall MHD, as d_i increases, changes from an elongated sheet geometry (Sweet-Parker type [8, 9]) to a more open X-point geometry (Petschek type [3]). However, recent fully kinetic simulations [3, 6] and EMHD-based treatments [2] have shown that elongated current sheets are also possible. To deepen the controversy, more recent particle-in-cell simulations [11] show spatial localization of the out-of-plane current to within a few d_i's of the X-line.

In an effort to shed more light on this issue, Shivamoggi [15] investigated whether the Hall effects favor the hyperbolicity of the magnetic field near a two-dimensional X-type magnetic neutral line. In this investigation, asymptotic solutions to a non-linear ordinary differential equation are used. In our investigation we shall determine if those approximations satisfy exact equations in order to give the solutions more credence.

ESSAY

For the purposes of our investigation, we shall use the same initial-value problem set up by Shivamoggi [15]. The system is two-fluid, quasi-neutral plasma near a two-dimensional X-type magnetic neutral line. In this system, the magnetic field is given by

$$\psi(x, y, t) = k\alpha(t)x^2 - \beta(t)y^2 \tag{1}$$

The solution to the system given in [15] is

$$\alpha(t) = e^{2(\gamma+\sigma Ct)}, \tag{2}$$

$$\beta(t) = e^{-2(\gamma+\sigma Ct)}, \tag{3}$$

where $\gamma(t)$ satisfies the following differential equation

$$\dot{\gamma}^2 + 2\sigma C\dot{\gamma} = [k^2 e^{4(\gamma+\sigma Ct)} + e^{-4(\gamma+\sigma Ct)}] - A, \tag{4}$$

with the initial conditions

$$t = 0 : \gamma = 0,\ \dot{\gamma} = \gamma_O,\ \alpha = \beta = 1, \tag{5}$$

where γ_0 and k are externally determined parameters and $\gamma_0 > 0$; $C > 0$; $k > 1$. The parameter k is taken to be greater than one so that the Lorentz force of the system is directed to maintain the prescribed initial stagnation-point flow. Using equation (4) with the initial conditions (5) yields

$$A = (k^2 + 1) - \gamma_O^2 - 2\sigma C\gamma_O. \tag{6}$$

Since the equations themselves do not seem to permit exact solutions, asymptotics can be used to approximate solutions for short and long times. Short and long times are compared to an Alfvén time τ_A, which may be taken to be γ_0^{-1}. For short time, equation (4) gives

$$\gamma(t) \approx \gamma_O t + (k^2 - 1)t^2, \tag{7}$$

while for long time, equation (4) gives

$$\gamma(t) \approx -\sigma Ct. \tag{8}$$

But do these approximations satisfy compatibility conditions like the plasma incompressibility condition and the exact invariant? The equation for incompressibility is given by

$$\frac{\dot{\alpha}}{\alpha} + \frac{\dot{\beta}}{\beta} = 0, \tag{9}$$

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while the invariant outlined in [15] is

(10) $$I = (\dot{\gamma} + \sigma C)^2 - (k^2\alpha^2 + \beta^2) = const.$$

Note that equations (2) and (3) have the following relationship

(11) $$\beta = \alpha^{-1}.$$

Substituting equation (11) into (9) yields

(12) $$\frac{\dot{\alpha}}{\alpha} + \frac{(\dot{\alpha^{-1}})}{\alpha^{-1}} = \frac{\dot{\alpha}}{\alpha} - \frac{\alpha^{-2}\dot{\alpha}}{\alpha^{-1}} = \frac{\dot{\alpha}}{\alpha} - \frac{\dot{\alpha}}{\alpha} = 0.$$

So the incompressibility of the plasma is satisfied regardless of γ, since α is non-zero. Plugging equations (2) and (3) into (4) and completing the square yields

(13) $$[k^2\alpha^2 + \beta^2] = A + \dot{\gamma}^2 + 2\sigma C\dot{\gamma} = A + (\dot{\gamma} + \sigma C)^2 - \sigma^2 C^2.$$

Substituting equation (13) into the invariant (10) gives

(14) $$(\dot{\gamma} + \sigma C)^2 - [A + (\dot{\gamma} + \sigma C)^2 - \sigma^2 C^2] = -A + \sigma^2 C^2 = \lambda^2 - k^2 - 1$$

where

(15) $$\lambda \equiv \gamma_o + \sigma C.$$

This shows that a proper γ solution that satisfies the ODEs will satisfy the invariant. For large *t*, plugging in equation (8) into (10) yields

(16) $$(-\sigma C + \sigma C)^2 - [k^2 e^{2(-\sigma Ct + \sigma Ct)} + e^{-2(-\sigma Ct + \sigma Ct)}] = -k^2 - 1$$

So the large time approximation for γ trivially satisfies the invariant. For small time, plugging equations (2), (3) and (7) into (10) gives

(17) $$I = (\lambda + 2(k^2 - 1)t)^2 - [k^2 e^{4(\lambda t + (k^2-1)t^2)}].$$

We must use the Taylor polynomials for α^2 and β^2 because exponentials and polynomials cannot be directly compared. Using equation (7) for γ, the second degree Taylor polynomials for α^2 and β^2 are the following

(18) $$\alpha_2^2 = 1 + 4\lambda t + (8\lambda^2 + 4(k^2 - 1))t^2,$$
$$\beta_2^2 = 1 - 4\lambda t + (8\lambda^2 - 4(k^2 - 1))t^2.$$

Substituting equation (7) and (18) into (10) yields

(19) $$I_2 = \lambda^2 - k^2 - 1 - 8\lambda^2(k^2 + 1)t^2,$$

where I_2 denotes the invariant with degree two Taylor polynomials replacing α^2 and β^2. This shows that for small time, the invariant is only satisfied up to $O(t)$; however, the invariant can be used to refine γ, making the following definition

$$I_m \equiv \dot{\zeta}^2 - (k^2\alpha_m^2 + \beta_m^2) = const, \tag{20}$$

where

$$\zeta(t) \equiv \gamma(t) + \sigma Ct, \tag{21}$$

and $\alpha^2{}_m$ and $\beta^2{}_m$ are the m^{th} degree Taylor Polynomials for α^2 and β^2 respectively. Differentiating I_m repeatedly with respect to time gives

$$\dot{I}_m = 2\dot{\zeta}\ddot{\zeta} - \frac{d}{dt}(k^2\alpha_m^2 + \beta_m^2) = 0,$$

$$\ddot{I}_m = 2\dot{\zeta}\zeta^{(3)} + 2\ddot{\zeta}^2 - \frac{d^2}{dt^2}(k^2\alpha_m^2 + \beta_m^2) = 0,$$

$$I_m^{(3)} = 2\dot{\zeta}\zeta^{(4)} + 4\ddot{\zeta}\zeta^{(3)} - \frac{d^3}{dt^3}(k^2\alpha_m^2 + \beta_m^2) = 0.$$

We can see the following pattern emerge

$$I_m^{(n)} = 2\dot{\zeta}\zeta^{(n+1)} + \sum_{i=1}^{P(n)}[a_i\zeta^{(j_i)}\zeta^{(k_i)}] - \frac{d^n}{dt^n}[k^2\alpha_m^2 + \beta_m^2] = 0 \tag{22}$$

where $\sum_{i=1}^{P(n)}[a_i\zeta^{(j_i)}\zeta^{(k_i)}]$ is subject to $n \geq 1$; $j_i + k_i = n + 2$; $1 < j_i, ki < n + 1$; $a_i, j_i, k_i \in N$; and $P(n)$ is the number of unique ways of accomplishing $j_i + k_i = n + 2$. If one does not like the sum, recall that

$$\frac{d^n}{dt^n}(\dot{\zeta}^2) = 2\dot{\zeta}\zeta^{(n+1)} + \sum_{i=1}^{P(n)}[a_i\zeta^{(j_i)}\zeta^{(k_i)}]. \tag{23}$$

Note that equation (22) is equivalent to zero only for a properly chosen γ. Let us treat the equation as if γ is written in the following form: $\gamma = \sum_{i=1}^{\infty} c_i t^i$. If γ is written in that form, then taking *n* derivatives of γ eliminates the first *n* terms of γ.

(24) $$\gamma^{(n)} = \frac{n!}{0!}c_n + \frac{(n+1)!}{1!}c_{n+1}t + \frac{(n+2)!}{2!}c_{n+2}t^2 + \ldots = \sum_{i=0}^{\infty}\frac{(n+i)!}{i!}c_{n+i}t^i.$$

If we evaluate equation (24) at t = 0 then only the constant term will remain

(25) $$\gamma^{(n)}(0) = n!c_n$$

Recall from equation (19) that $\gamma = \gamma_0 t + (k^2 - 1)t^2$ does not return a constant if directly plugged into I_m. We can intuitively understand that c_3 should be non-zero. We can effectively solve equation (22) for $\zeta^{(n+1)}$ under the assumption that a proper m is used (meaning that $\alpha^2{}_m$ and $\beta^2{}_m$ are not functions using $\zeta^{(n+1)}$),

(26) $$\zeta^{(n+1)} = \left(-\sum_{i=1}^{P(n)}[a_i\zeta^{(j_i)}\zeta^{(k_i)}] + \frac{d^n}{dt^n}[k^2\alpha_m^2 + \beta_m^2]\right)\Big/\left(2\dot{\zeta}\right).$$

In order to solve for the missing c_3 term, we can use equation (25) and evaluate equation (26) using t = 0 and n = 2. (Note that higher order derivatives of ζ are equal to the higher order derivatives of γ.) Using $m = n$ results in $\alpha^2{}_m$ and $\beta^2{}_m$ only using $\zeta^{(n)}$ and below since $\alpha^2 = e^{4\zeta}$ and $\beta^2 = e^{-4\zeta}$.

(27) $$\zeta^{(3)}(0) = 3!c_3 = \left(-\sum_{i=1}^{P(2)}[a_i\zeta^{(j_i)}(0)\,\zeta^{(k_i)}(0)] + \left[\frac{d^2}{dt^2}[k^2\alpha_2^2 + \beta_2^2]\right|_{t=0}\right)\Big/\left(2\dot{\zeta}(0)\right).$$

We may generalize equation (27) for arbitrary order and prescribe that it is used only when $c_1, c_2, \ldots, c_n$ are known (c_{n+1}, c_{n+2}, are all 0 at the time of evaluation of $I_n^{(n)}$) and $n \geq 2$, which results in

(28) $$c_{n+1} = \frac{-I_n^{(n)}(0)}{2(n+1)!\dot{\zeta}(0)} = \frac{-I_n^{(n)}(0)}{2\lambda(n+1)!}.$$

Equation (28) is valid because when c_{n+1} is zero, $\zeta^{(n+1)}$ is zero, which in turn causes equation (22) to reduce to

(29) $$I_n^{(n)}(0) = \sum_{i=1}^{P(n)}[a_i\zeta^{(j_i)}(0)\,\zeta^{(k_i)}(0)] - \left[\frac{d^n}{dt^n}[k^2\alpha_n^2 + \beta_n^2]\right|_{t=0}$$

So by recursively applying the formula for c_{n+1} while incrementing n by 1, we can solve for the unknown coefficients of γ and create a higher order numerical solution to the coupled non-linear ordinary differential equations for small time.

Now that we have derived an effective method for generating higher degree approximations for γ, we naturally want an equation that quantifies the approximations' quality. Because γ is a polynomial, I_m is composed of a linear combination of polynomials. As such, I_m can be written in the following form: $I_m = \sum_{i=0}^{\infty} b_i t^i$. As in the treatment of γ in equations (24) and (25), we can take n derivatives of I_m and evaluate the result at t = 0 to isolate the constant.

$$I_m^{(n)}(0) = n!b_n. \quad (30)$$

Because I_m is supposed to be a constant for a properly chosen γ, any non-zero b_n for $n \geq 1$ is considered to be an error. As seen in equation (19), plugging an $O(t^2)$ γ approximation into I_2 led to an error of $O(t^2)$. Because equation (28) eliminates one order of error every time it recurses, plugging in γ to $O(t^n)$ into the invariant will return an error of $O(t^n)$; therefore, to receive the leading order error from plugging γ into the invariant, one can use the following formula:

$$E_n = b_n t^n = \frac{I_n^{(n)}(0)\,t^n}{n!} \quad (31)$$

E_n should be used under the same conditions that apply to the c_{n+1} equation. That is, only when $c_1, c_2, \ldots, c_n$ are known ($c_{n+1}, c_{n+2}, \ldots$ are all 0 at the time of evaluation of $I_n^{(n)}$) and $n \geq 2$. If we use E_n when c_{n+1} is its proper non-zero value, then E_n will return 0. E_n is used to visually and numerically determine the accuracy of small time γ approximations.

Now that we have derived a way to refine the small time approximations for γ to higher degrees and have determined its respective accuracy, let us get a better appreciation for its process. Figure 1 shows the absolute error of successive refinements of γ with λ = k = 1.5. We can see in Figure 1 that depending on which order of approximation for γ is used there is a point at which the approximation becomes too inaccurate to be useful. The amount of time where the error is almost zero increases for higher degree approximations. The range of time from $t = 0$ to the point of inaccuracy is a restriction of γ denoted by $D = [0; t^*]$. The point of inaccuracy is defined to be the point where the error has a magnitude equal to 0.001. Not only does the degree of γ affect the value of t^*, but the choice of the constants γ_0, σ, C, and k also affect t^*. So, for the purpose of determining their affect on t^*, we shall treat them as variables rather than constants and keep the order of γ fixed. To reduce the visual dimensionality of E_n from six to four, we shall use equation (15). Now

(32) $$E_n = E_n(t, \lambda, k).$$

Because we cannot readily see 4-dimensional data in a single picture, we must make discrete slices of the 4-D surface to turn it into many 3-D surfaces. Slices along the time axis are used. However, this report is on paper, which is a 2-D surface, and 3-D images are hard to discern when projected onto paper. To reduce the dimension to two, we shall make a planar slice normal to the error axis. To extract those points from $E_n(t, \lambda, k)|_{t=t^*}$ we can plot the intersection of the 3-D surface with the plane normal to the error axis and containing the point $(t, \lambda, k, E_n) = (t^*, 0, 0, 0.001)$. The point of inaccuracy is now a curve in the (λ, k) plane. Performing the aforementioned planar slice for various t^* values and using a 10th order γ results in a contour map (see Figure 2). We can see in Figure 2 that as the values of λ and k increase, the value of t^* decreases, and as $\lambda \to 0$, $t^* \to \infty$ because $E_n \equiv 0$ when $\lambda = 0$. We may vary the method in which E_n is partitioned to create contours in other planes (see Figure 3 and Figure 4). We can also solve the differential equation using the Runge-Kutta numerical method. In doing so, we find a finite-time singularity for moderate values of Hall parameter σ, which is tied in our constant λ (see Figure 5).

FIGURE 1. ABSOLUTE ERROR OF THE SMALL TIME APPROXIMATION FOR γ WITH $\lambda = k = 1.5$ AND VARYING DEGREE γ

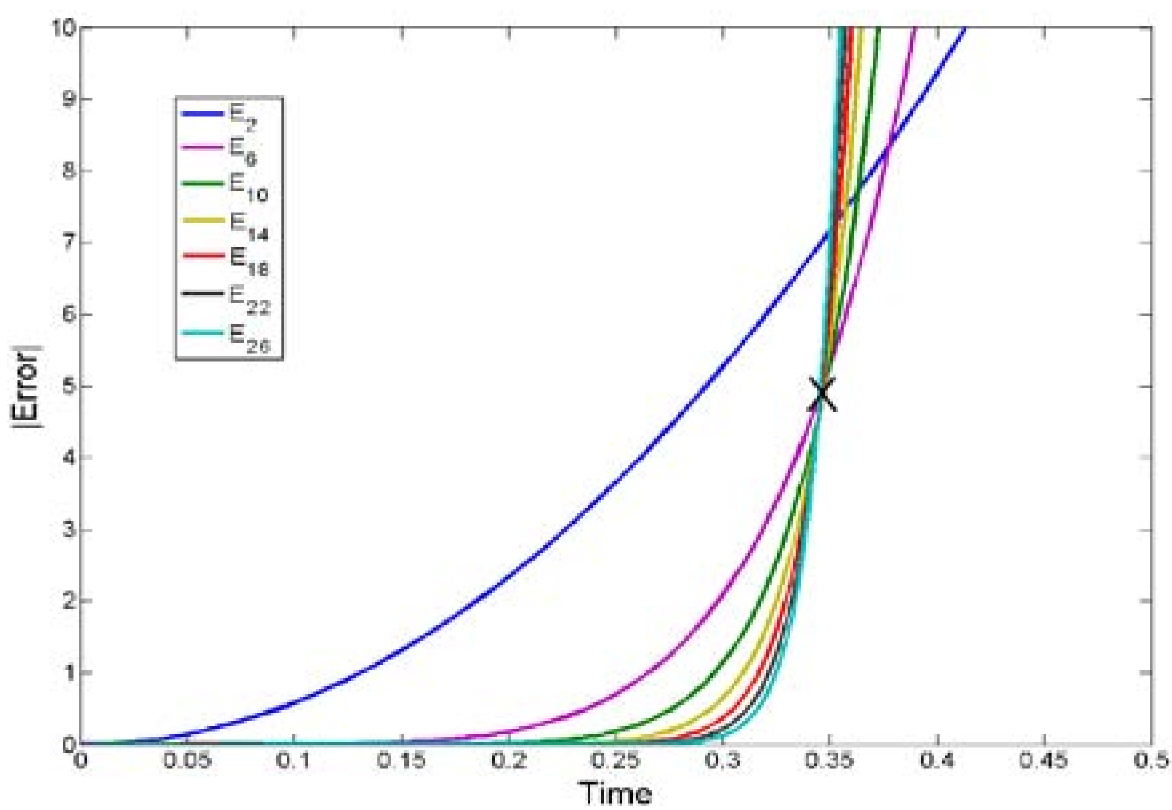


FIGURE 2. CONTOUR MAP SHOWING THE CHANGES OF THE LENGTH OF TIME FOR WHICH THE SMALL TIME APPROXIMATION FOR γ IS ACCURATE FOR VARYING VALUES OF λ AND k WITH γ FIXED AS A 10TH DEGREE POLYNOMIAL

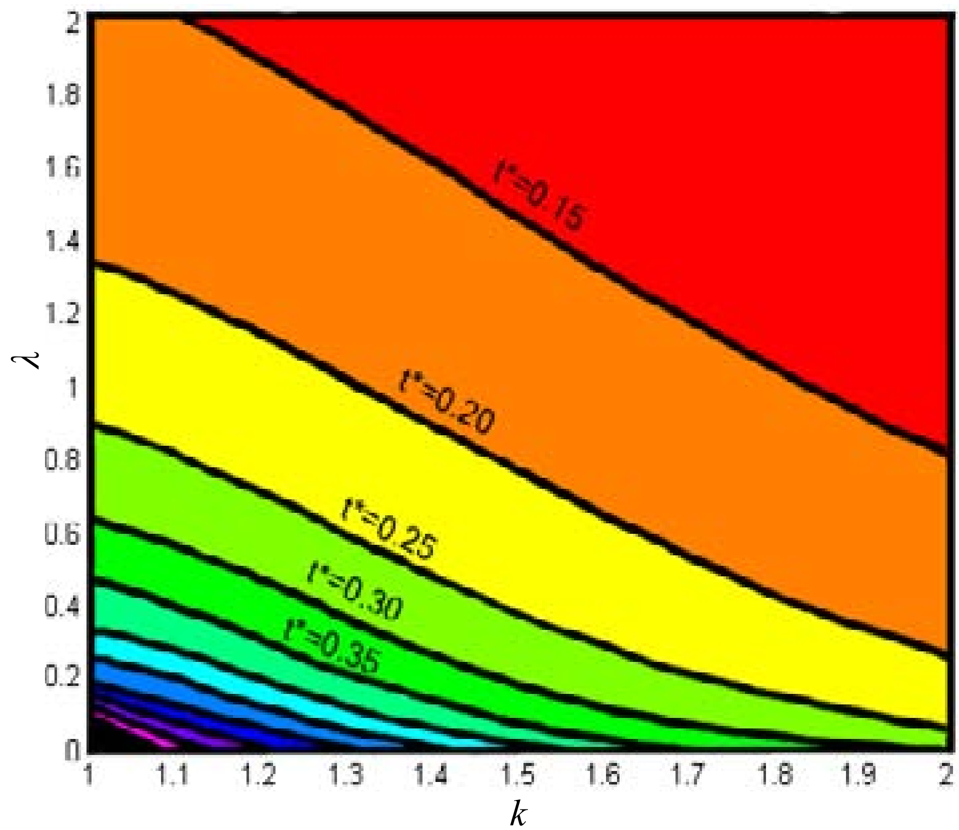


FIGURE 3. CONTOURS IN THE (λ, t*) PLANE WITH γ FIXED AS A 10TH DEGREE POLYNOMIAL

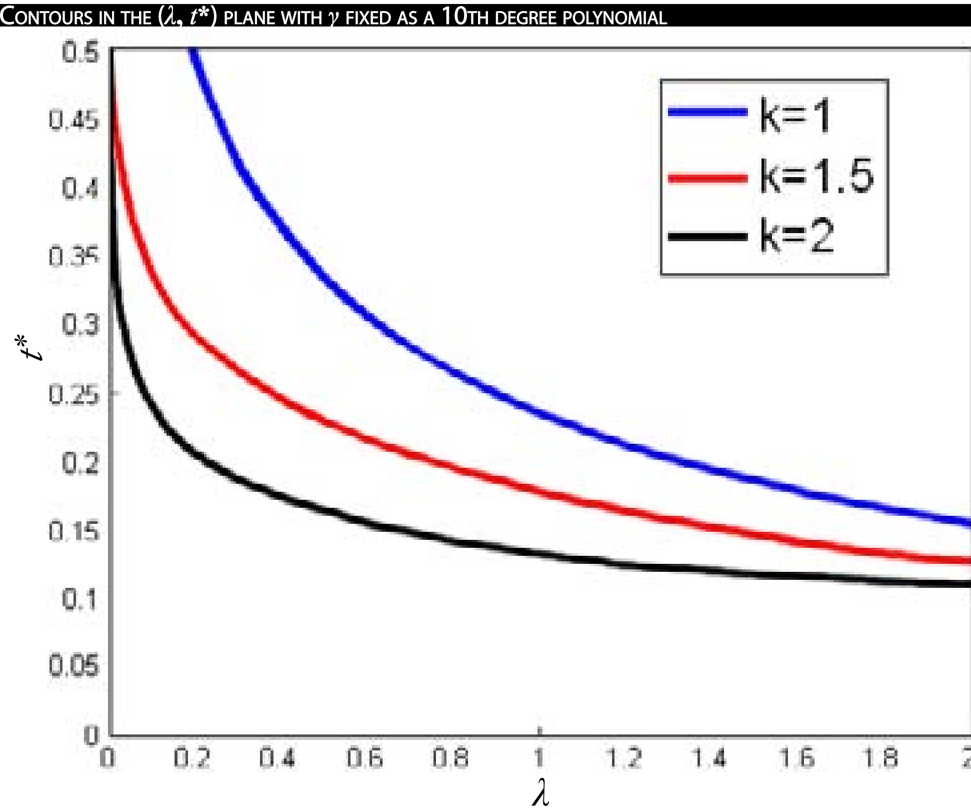


FIGURE 4. CONTOURS IN THE (k, t^*) PLANE WITH γ FIXED AS A 10TH DEGREE POLYNOMIAL

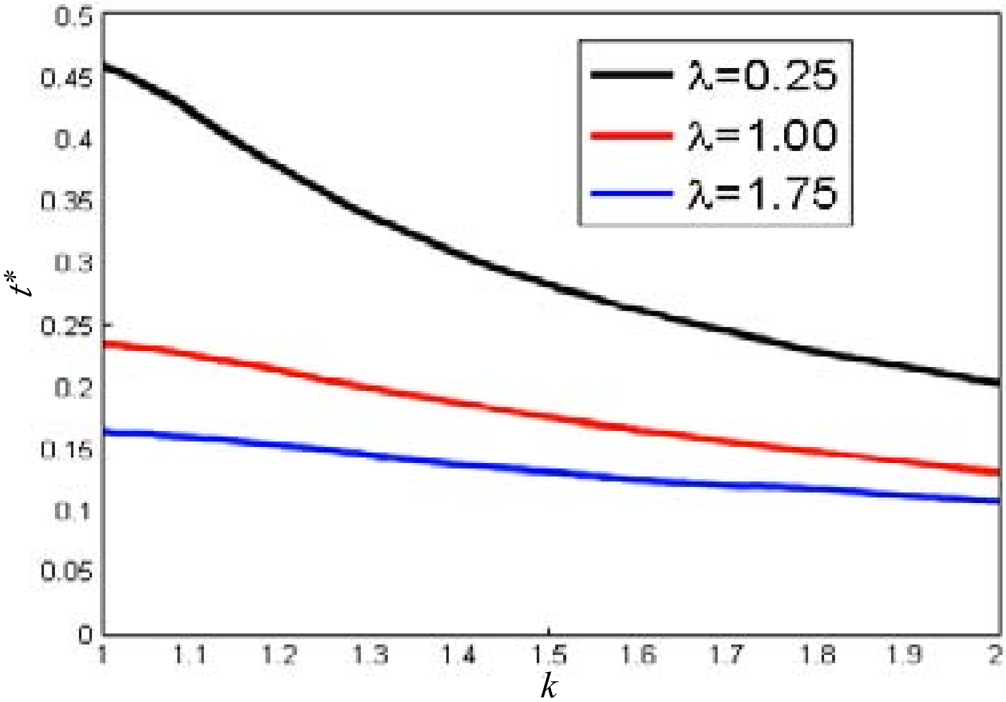
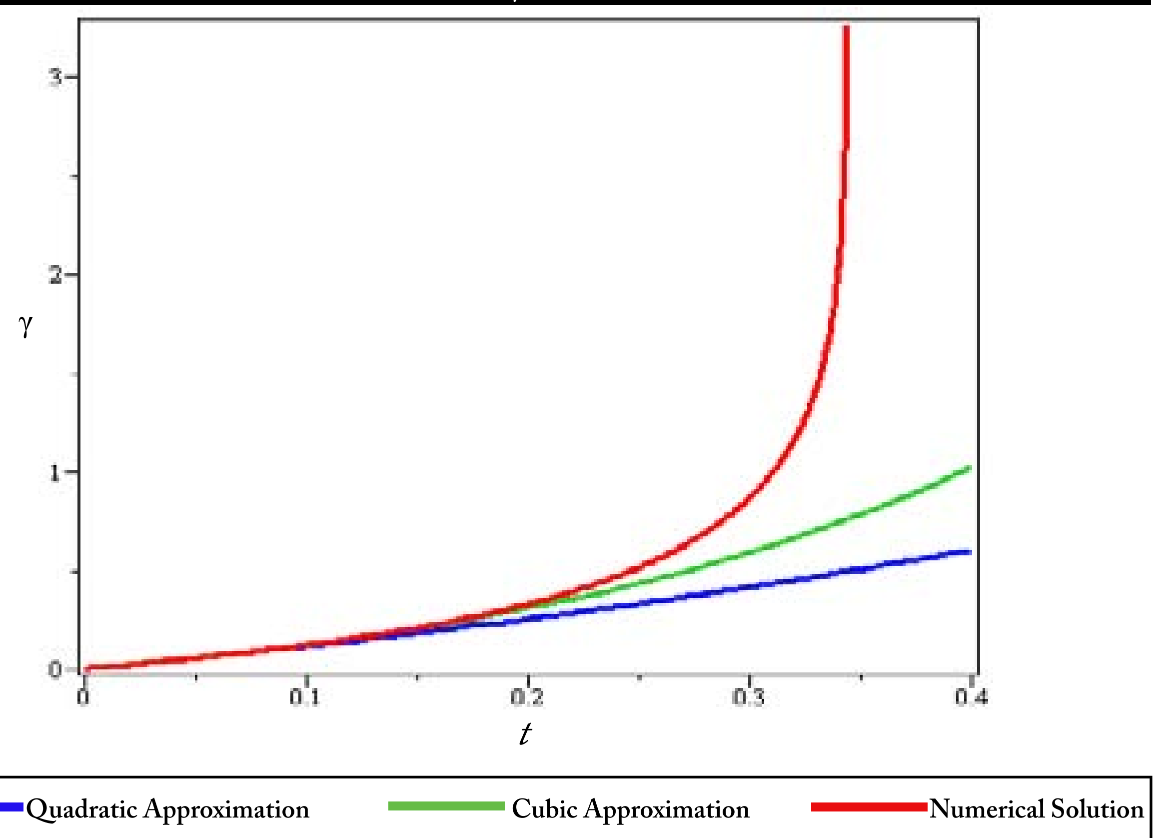


FIGURE 5. POLYNOMIAL APPROXIMATIONS AND NUMERICAL SOLUTION TO γ FOR $\lambda = k = 1.5$



To determine the time at which the singularity occurs, we introduce the transformation

(33) $$-\ln(u(t)) = 4\gamma(t) + 4\sigma Ct, \; u(0) = 1,$$

which reduces (4) to

(34) $$\dot{u}^2 = 16k^2u + 16mu^2 + 16u^3,$$

where for simplicity we define the constant

(35) $$m \equiv (\gamma_0 + C)^2 - k^2 - 1 .$$

Equation (34) admits the implicit solution

(36) $$t \pm \frac{1}{4}\int_1^{u(t)} \frac{dz}{\sqrt{k^2 z + mz^2 + z^3}} = 0$$

where we have used the initial condition $u(0) = 1$ to obtain the lower point of integration. Note that, when γ becomes singular, this corresponds to $u \to 0$. So, if $t = t^+$ is a singular point, we have that $u(t^+) = 0$. Placing this assumption into (36), we find that

(37) $$t^+ \pm \frac{1}{4}\int_1^{0} \frac{dz}{\sqrt{k^2 z + mz^2 + z^3}} = 0$$

We are concerned with $t \geq 0$, so we take the positive of the two solutions, obtaining

(38) $$t^+ = \frac{1}{4}\int_0^{1} \frac{dz}{\sqrt{k^2 z + mz^2 + z^3}} .$$

This gives the first singularity in the region $t \geq 0$. Equation (38) is only accurate in the parameter regime $k > 1$, $\sigma C > 0$, $\gamma_0 > 0$. It is interesting to note that for $\lambda = k = 1.5$, $t^+ \approx 0.3439$, which appears to be the limit of the polynomial approximations' accuracy as $n \to \infty$ and thus $\lim_{n\to\infty} t^*(n) = t^+$. This point is marked with a black X in Figure 1.

Now that we have established that a singularity for a certain parameter regime exists, we may instead use a different model for the behavior of γ based on [14], which studies the ideal MHD model where $\sigma = 0$. In [14], the differential equation is

$$\dot{\mu}^2 = k^2e^{4\mu} + e^{-4\mu} - C \qquad (39)$$

and its large time approximation is given by

$$\mu(t) \approx \frac{-1}{2}\ln(2k(t_0 - t)) \qquad (40)$$

where t_0 is the time when the singularity occurs. Note that if we use our transformation (21) on (4), we will get an identical differential equation; therefore, we may use the inverse of the transform and create the long time approximation

$$\gamma(t) \approx \frac{-1}{2}\ln(2k(t^+ - t)) - \sigma Ct . \qquad (41)$$

However, this approximation has an error of $\frac{-1}{2}\ln(2kt^+)$ at $t = 0$ and an approximately quadratic error decay until $t = t^+$. We thus introduce a quadratic correction term to equation (41) to reduce the error.

$$\gamma(t) \approx \frac{-1}{2}\ln(2k(t^+ - t)) - \sigma Ct + \frac{1}{2(t^+)^2}\ln(2kt^+)(t^+ - t)^2 \qquad (42)$$

Although this approximation has $O(t)$ error when plugged into the invariant, its absolute error when compared to the numerical solution to the differential equation is bounded by

$$E \leq \frac{1}{4}\left|\ln(2kt^+)\right|, \qquad (43)$$

and therefore is very accurate for large values of λ and k.

But what of our original large time asymptotic approximation for γ, equation (8)? The linear behavior is only observed in a certain parameter regime where $\gamma_0 = -\sigma C$. With this condition, the Hall term successfully prevents the finite-time singularity, and the linear approximation is accurate. This conincides with our previous note that $t^* \to \infty$ as $\lambda \to 0$ and thus the singularity never occurs.

CONCLUSION

In our investigation into Hall MHD near a 2D X-type neutral point, we have tested the accuracy and validity of the asymptotic solutions found in [15] by checking to see if they satisfy exact equations. We have found that these solutions satisfy the incompressibility condition. The asymptotic solution for long time trivially satisfies the exact invariant. However, the small time asymptotic solution only satisfies the invariant up to $O(t)$. Further investigation into the nature of the invariant revealed that it could be used to recursively refine the small time solution to higher degree polynomials. Despite having an arbitrary order of precision, the refined solution is only valid for a short time interval—a length which is dependent on the externally determined parameters of the system. Unlike the ideal MHD model investigated in [14], the Hall MHD model has the ability to not exhibit a finite-time singularity. This allows the magnetic field lines to stay in a more open hyperbolic configuration and prevents the current-density blow-up and plasma collapse at long times in Hall MHD.

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REFERENCES

[1] D. Biskamp, *Magnetic reconnection in plasmas*, Cambridge University Press, 2000.

[2] L. Chacón, A. N. Simakov, and A. Zocco, *Steady-state properties of driven magnetic reconnection in 2d electron magnetohydrodynamics*, Physical Review Letters **99** (2007), 235001.

[3] W. Daughton, J. Scudder, and H. Karimabadi, *Fully kinetic simulations of undriven magnetic reconnection with open boundary conditions*, Physics of Plasmas **13** (2006), 072101.

[4] J. C. Dorelli and J. Birn, *Whistler-mediated magnetic reconnection in large system: Magnetic flux pileup and the formation of thin current sheets*, Journal of Geophysical Research **108** (2003), 1133.

[5] Richard Fitzpatrick, *Scaling of forced magnetic reconnection in the Hall-magnetohydrodynamic Taylor problem*, Physics of Plasmas **11** (2004), no. 3, 937-46.

[6] H. Karimabadi, W. Daughton, and J. Scudder, *Multi-scale structure of the electron diffusion region*, Geophysical Research Letters **34** (2007), 235001.

[7] D. A. Knoll and L. Chacón, *Coalescence of magnetic islands, sloshing, and the pressure problem*, Physics of Plasmas **13** (2006), no. 3, 032307, 7.

[8] E. N. Parker, *Sweet's mechanism for merging magnetic fields in conducting fluids*, Journal of Geophysical Research **62** (1957), 509-20.

[9] Petchek, *ASS/NASA symposium on the physics of solar flares*, p. 425, NASA, Washington, DC, 1964.

[10] B. N. Rogers, R. E. Denton, J. F. Drake, and M. A. Shay, *Role of dispersive waves in collisionless magnetic reconnection*, Physical Review Letters **87** (2001), 195004.

[11] M. A. Shay, J. F. Drake, and M. Swisdak, *Two-scale structure of the electron dissipation region during collisionless magnetic reconnection*, Physical Review Letters **99** (2007), 155002.

[12] M. A. Shay, J. K. Drake, B. N. Rogers, and R. E. Denton, *The scaling of collisionless, magnetic reconnection for large systems*, Geophysical Research Letters **26** (1999), 2163-66.

[13] B. K. Shivamoggi, *Magnetohydrodynamic theories of magnetic field reconnection*, Physics Report **127** (1985), no. 2, 99-184.

[14] B. K. Shivamoggi, *Current-sheet formation near a hyperbolic magnetic neutral line*, Springer Lecture Notes in Physics **571** (2001), 381-88.

[15] B. K. Shivamoggi, *Hall magnetohydrodynamics near an x-type magnetic neutral line*, Europhysics Letters **85** (2009), 25001.

[16] B. U. O. Sonnerup, *Solar system plasma physics*, p. 45, North-Holland, 1979.

[17] P. A. Sweet, *Electromagnetic phenomena in cosmic physics*, p. 123, Cambridge University Press, 1958.